

## ANALYSIS AND DESIGN OF IDEAL NON SYMMETRICAL COUPLED MICROSTRIP DIRECTIONAL COUPLERS

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## ABSTRACT

Ideal directional couplers consisting of coupled asymmetric lines in an inhomogeneous medium have been realized by equalizing the inductive and capacitive coefficients of coupling. Analysis, design procedure and examples of such high directivity asymmetric couplers consisting of various planar and layered structures are presented together with the experimental results for 6 db couplers on Alumina.

## INTRODUCTION

This paper deals with the analysis and design of directional couplers consisting of non symmetrical coupled lines in an inhomogeneous medium such as asymmetric coupled microstrips. The theory and design of directional couplers consisting of symmetrical coupled lines in homogeneous or inhomogeneous medium and asymmetric coupled lines in a homogeneous medium is well known [e.g., 1-6]. The study of the directional couplers consisting of asymmetric coupled lines in an inhomogeneous medium where the normal mode velocities are not equal has, however, been limited to the analysis of the four ports with non mode converting terminations [7] and a general analysis procedure for computing the scattering parameters of the four port [8].

In this paper the procedure to design ideal couplers having all matched ports and perfect directivity consisting of non symmetrical coupled lines in an inhomogeneous medium is reported. The analytical results have been verified numerically by computer simulation of the idealized distributed parameter systems and experimentally by comparing the scattering parameters of directional couplers designed with and without the techniques for improving coupler performance presented in the paper.

## THEORY AND COUPLER DESIGN

The general uniformly coupled line structure can be characterized by the equivalent self and mutual series impedance and shunt admittance of the lines. For the loss less case the coupled transmission line equations are :

$$\frac{d}{dz} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = -j\omega \begin{bmatrix} L_{11} & L_m \\ L_m & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{and}$$

$$\frac{d}{dz} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = -j\omega \begin{bmatrix} C_{11} & C_m \\ C_m & C_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (1)$$

where,  $C_{11}$ ,  $C_m$  and  $C_{22}$  are the self and mutual capacitances per unit length of the coupled line system and  $L_{11}$ ,  $L_m$  and  $L_{22}$  are the corresponding inductances per unit length. For dispersive or non quasi TEM systems these are the equivalent values derived from the normal mode parameters [9,11].

The coupling mechanism in the coupled line structure can be investigated in terms of either the coupled modes or the normal modes of the system [6,9]. Both these techniques are well known and have been used to study circuit elements including couplers. Following the normal mode analysis, the voltages and currents at the four ports (Fig. 1) are given by,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ R_c & R_c & R_\pi & R_\pi \\ R_c e^{-j\theta_c} & R_c e^{j\theta_c} & R_\pi e^{-j\theta_\pi} & R_\pi e^{j\theta_\pi} \\ e^{-j\theta_c} & e^{j\theta_c} & e^{-j\theta_\pi} & e^{j\theta_\pi} \end{bmatrix} \begin{bmatrix} V_{1c+} \\ V_{1c-} \\ V_{1\pi+} \\ V_{1\pi-} \end{bmatrix} \quad \text{and} \quad (2)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{c1} & -Y_{c1} & Y_{\pi1} & -Y_{\pi1} \\ R_c Y_{c2} & -R_c Y_{c2} & R_\pi Y_{\pi2} & -R_\pi Y_{\pi2} \\ R_c Y_{c2} e^{-j\theta_c} & -R_c Y_{c2} e^{j\theta_c} & R_\pi Y_{\pi2} e^{-j\theta_\pi} & -R_\pi Y_{\pi2} e^{j\theta_\pi} \\ Y_{c1} e^{-j\theta_c} & -Y_{c1} e^{j\theta_c} & Y_{\pi1} e^{-j\theta_\pi} & -Y_{\pi1} e^{j\theta_\pi} \end{bmatrix} \begin{bmatrix} V_{1c+} \\ V_{1c-} \\ V_{1\pi+} \\ V_{1\pi-} \end{bmatrix}$$

$R_c$  and  $R_\pi$  are the mode voltage ratios for the two normal modes,  $Y_{c1}$ ,  $Y_{\pi1}$ ,  $Y_{c2}$ ,  $Y_{\pi2}$  are the partial mode admittances of the two lines for the two modes,

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$\theta_c = \beta_c \ell$  and  $\theta_\pi = \beta_\pi \ell$ .  $\beta_c$  and  $\beta_\pi$  are the phase constants of the two normal modes and  $\ell$  is the length of the coupled section.

The normal mode parameters are given by [9],

$$\beta_{c,\pi} = \frac{\omega}{\sqrt{2}} \left[ L_{11}C_{11} + L_{22}C_{22} - 2L_mC_m \right. \\ \left. \pm \sqrt{(L_{11}C_{11} - L_{22}C_{22})^2 + 4(L_mC_{11} - L_{22}C_m)(L_mC_{22} - L_{11}C_m)} \right]^{1/2} \quad (3a)$$

$$R_{c,\pi} = \frac{L_{22}C_{22} - L_{11}C_{11} \pm \sqrt{(L_{11}C_{11} - L_{22}C_{22})^2 + 4(L_mC_{11} - L_{22}C_m)(L_mC_{22} - L_{11}C_m)}}{2(L_mC_{22} - L_{11}C_m)} \quad (3b)$$

$$Z_{c1,\pi1} = \frac{\omega}{\beta_{c,\pi}} \left( L_{11} - \frac{L_m}{R_{\pi,c}} \right) = \frac{1}{Y_{c1,\pi1}} \quad (3c)$$

$$Z_{c2,\pi2} = -R_c R_\pi Z_{c1,\pi1} = \frac{1}{Y_{c2,\pi2}} \quad (3d)$$

The amplitude constants are determined by the terminal conditions which for the terminated four port are  $V_2 = -Z_2 I_2$ ,  $V_3 = -Z_3 I_3$  and  $V_4 = -Z_4 I_4$ .

It is seen that four port can be matched ( $S_{ii} = 0$  for  $i = 1, 2, 3$  and  $4$ ) resulting in infinite directivity when the capacitive coefficient of coupling is made exactly equal to the inductive coefficient of coupling, i.e.,  $K_c = K_\ell$  where  $K_c = C_m/\sqrt{C_{11}C_{22}}$  and  $K_\ell = L_m/\sqrt{L_{11}L_{22}}$  and the four port is properly terminated. It is seen that for  $K_\ell = K_c$ , the partial line mode impedances are interrelated as  $Z_{c1} = -Z_{\pi1} = \sqrt{L_{11}C_{11}}$  and  $Z_{c2} = -Z_{\pi2} = \sqrt{L_{22}C_{22}}$ . Substitution of these expressions in Eq. (2) with terminations  $Z_1 = Z_{c1}$  and  $Z_2 = Z_{\pi2}$  leads to the ideal coupler transformer having a scattering matrix of the four port as given by,

$$S_{11} = S_{22} = S_{33} = S_{44} = S_{13} = S_{31} = S_{24} = S_{42} = 0 \quad (4a)$$

$$S_{12} = S_{21} = S_{34} = S_{43} = \frac{-\chi \left( 1 - e^{j(\beta_c + \beta_\pi)\ell} \right)}{\left( 1 - \chi^2 e^{j(\beta_c + \beta_\pi)\ell} \right)} \quad (4b)$$

$$S_{23} = S_{32} = \frac{(1 - \chi^2) e^{-j\beta_c \ell}}{\left( 1 - \chi^2 e^{-j(\beta_c + \beta_\pi)\ell} \right)} \quad (4c)$$

$$S_{14} = S_{41} = \frac{(1 - \chi^2) e^{j\beta_\pi \ell}}{\left( 1 - \chi^2 e^{-j(\beta_c + \beta_\pi)\ell} \right)} \quad (4d)$$

$$\text{with } \chi = \sqrt{R_\pi/R_c}.$$

The above expressions lead to a maximum coupling of  $2\chi/(1+\chi^2)$  and fractional 3 db bandwidth  $= 2/\pi \cdot [\cos^{-1}(2\chi/(1+\chi^2))]$ . The values of  $\chi$  (or  $1/\chi$ ) corresponding to 3 db, 6 db and 10 db couplers are .1715, .0718 and .0265, respectively. It should be noted that the above expressions are valid only when the ideal coupler conditions are satisfied, the structure is non symmetrical and the normal mode velocities or the eigenvalues are distinct. It is seen that  $R_c$  and  $R_\pi$  are extremely sensitive to changes in the self and mutual line constants near the ideal coupler condition values. In general, the scattering parameters are easily computed from the immittance matrix of the four port by using standard expressions such as,

$$[S] = \{ [Z] - [Z_t] \} \cdot \{ [Z] + [Z_t] \}^{-1} \quad (5)$$

where,  $[Z]$  is the impedance matrix of the four port as given in [9] and  $[Z_t]$  is the diagonal terminating impedance matrix.

Given the two terminating impedances, the coupling, the center frequency and the desired isolation or directivity, both edge and broadside coupled inhomogeneous structures can be physically realized. Examples of the edge coupled line structure are shown in Figure 1. For the case of coupled microstrips the capacitive coefficient of coupling is always less than the inductive one unless the medium is modified to enhance it in the same manner as done for symmetric coupled lines. Among the many techniques that can be used to increase  $K_c$  include the use of an overlay medium, composite substrate including the suspended substrate, uniaxial medium, a parallel slot or a tuning septum in the ground plane and external compensation with lumped capacitors.

Structures can be designed by using the quasi TEM normal mode parameters or the full wave parameters by using available numerical techniques [e.g., 10-14]. We have used the generalized spectral domain technique with both the global and the local basis approximations to compute the normal mode parameters of these structures. Scattering parameters for the four port can be computed for the ideal or the near ideal case in terms of these normal mode parameters. The structures can be designed either by tuning the input dimensions while monitoring the normal mode and the scattering parameters or by standard optimizing routines.

## RESULTS

Figure 2 demonstrates the variation in the capacitive coefficient of coupling and the normal mode velocity for a two layer substrate medium. It is seen that the ideal coupler conditions are satisfied for certain ratios of  $h_1/h_2$ . The ideal coupler geometry and performance is found to be quite sensitive to the variations in structure geometry, the terminating impedances and the effects of dispersion in the normal mode parameters. However, stable enhanced couplers have been designed by synthesizing the structure geometry required to meet desired specifications for coupling, matching and directivity for a given frequency band. Figure 3 shows the computed results

for two cases of nominal 6 db, 50  $\Omega$  to 75  $\Omega$  coupler transformers. These are a suspended substrate structure and a microstrip structure with a ground plane slot. The coupling and isolation for an edge coupled microstrip coupler on a simple substrate are included here to demonstrate the improvement in the coupler performance when the suspended substrate or the ground plane slot is used to equalize the capacitive and the inductive coefficient of coupling.

Two sets of coupled microstrip structures were also fabricated on 15 mil Alumina substrate. One set of four ports were designed and fabricated on simple substrate without any compensating techniques described in the paper and the other coupler was designed with a ground plane slot to enhance the coupler directivity. The theoretically computed values for coupling and directivity are shown in Figure 3b and the measured data is shown in Figure 4. The agreement between the two and the improvement in the coupler performance is seen to be more pronounced after the effects of short tapered sections, included with the input and output of the high impedance lines to facilitate measurements in a 50  $\Omega$  environment, are extracted.

In conclusion, the analysis and the design procedure for ideal non symmetrical edge coupled microstrip directional couplers is presented in the paper. Such hybrids offer additional flexibility in the design of single section, multi-section as well as asymmetric non uniform couplers with improved bandwidth and directivity.

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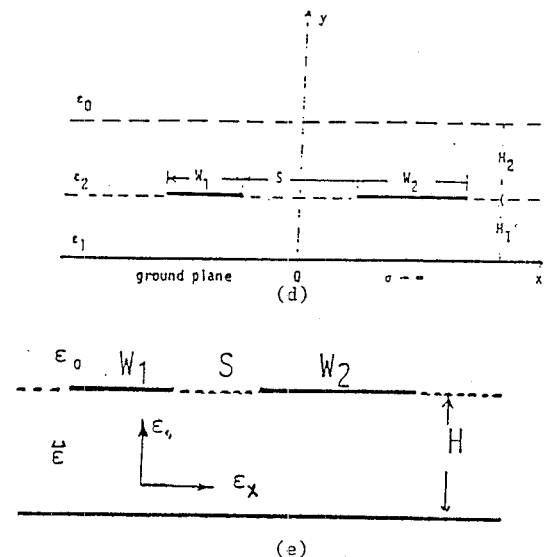
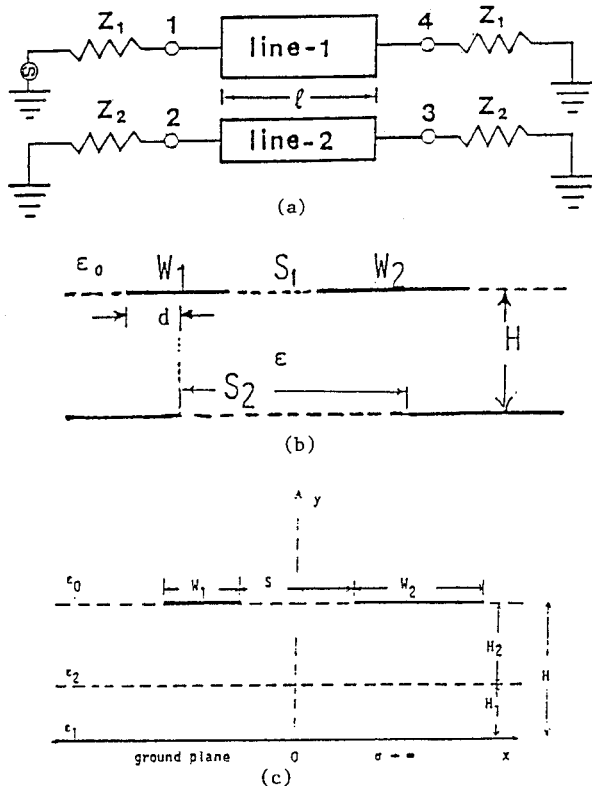


Figure 1 (a) Schematic of the asymmetric coupled line four port  
 (b) Coupled microstrips with a ground plane slot. (c) Coupled microstrips with a layered substrate. For suspended substrate lines  $\epsilon_1 = \epsilon_0$ . (d) Coupled microstrips with an overlay medium  
 (e) Coupled microstrips on a uniaxial medium.

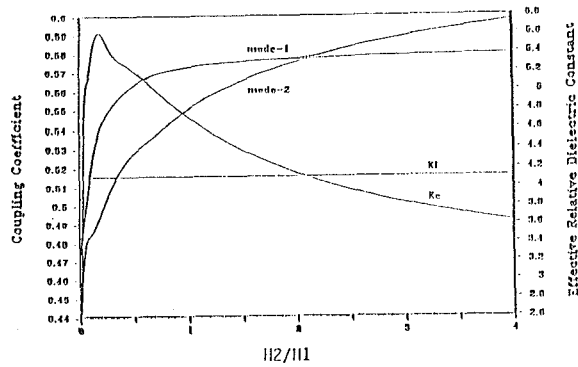


Figure 2. Coupling coefficients and mode velocities for a typical asymmetric coupled line structure on a two layer medium.  $\epsilon_1 = 4\epsilon_0$ ;  $\epsilon_2 = 9.8\epsilon_0$ ;  $W_1 = 2W_2 = 6S$ ;  $W_1/(H_1 + H_2) = 0.6$

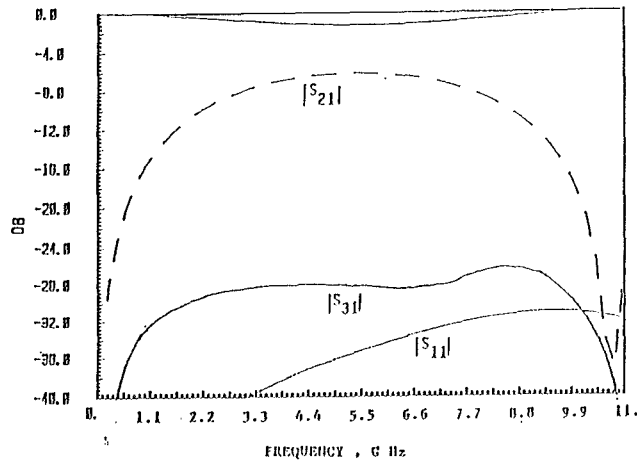
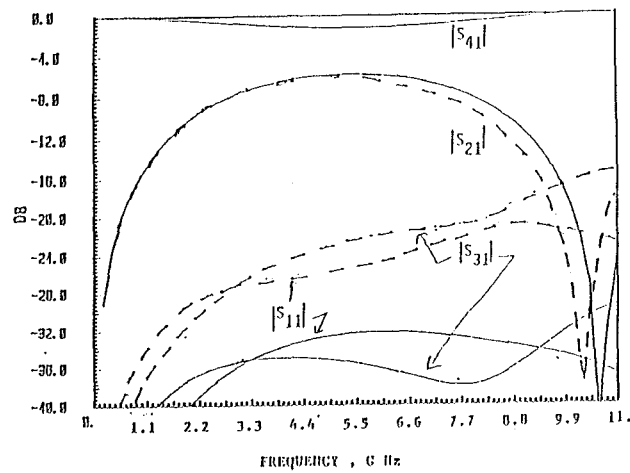


Figure 3. Computed scattering parameters for the 50  $\Psi$  to 75  $\Psi$  6 DB coupler transformers. (a) — microstrips with the ground plane slot  $W_1/H = .27$ ,  $W_2/H = .9$ ,  $S_1/H = .12$ ,  $S_2/H = 1.8$ ,  $d/H = .15$ ,  $\epsilon = 9.8$  --- without the slot  $W_1/H = .24$ ,  $W_2/H = .51$ ,  $S_1/H = .09$ ,  $\epsilon = 9.8$  (b) Coupled microstrips on a suspended substrate. (Fig. 1c  $\epsilon_{r1} = 1$ )  $W_1/H = .28$ ,  $W_2/H = .86$ ,  $S/H = .1$ ,  $H = H_1 + H_2$ ,  $\epsilon_{r2} = 9.8$ ,  $H_2/H_1 = 19$

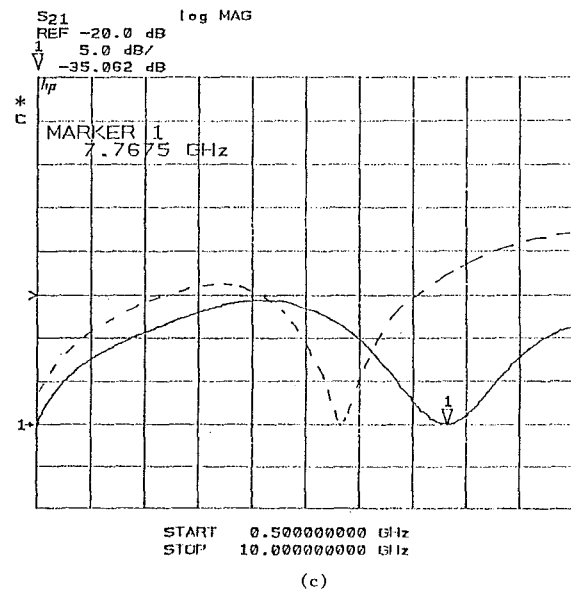
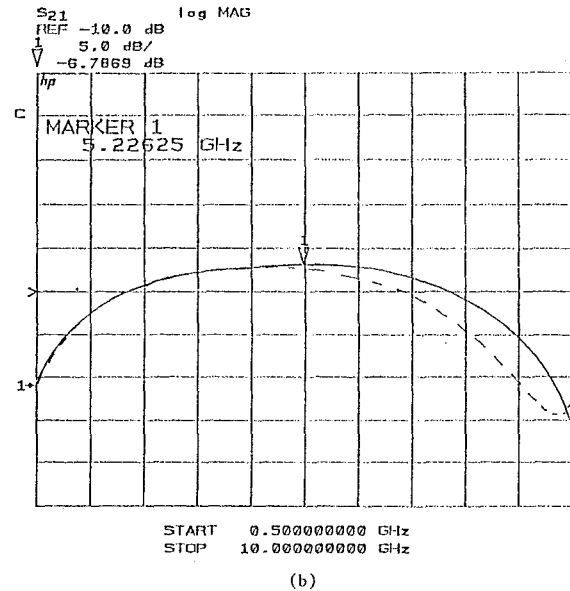
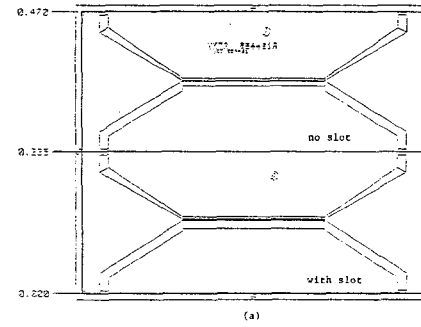


Figure 4. Experimental results (a) Coupler layout. Dimensions for the coupler are the same as that in figure 3a (b) Measured coupling,  $S_{21}$  (c) Measured isolation,  $S_{31}$  Solid curves for coupler with the ground plane slot and the dashed curves are for coupler without the slot.